2 MJ 60704

THREE YEAR B.Sc. (CBCS) DEGREE EXAMINATION, APRIL/MAY 2024.

SECOND SEMESTER

Mathematics – Major

Paper IV — ANALYTICAL SOLID GEOMETRY

(w.e.f 2023–24 Admitted Batch)

Time : Three hours

Maximum : 70 marks

(No additional sheet will be supplied)

SECTION A — ($5 \times 4 = 20$ marks)

Answer any FIVE of the following.

1. Find the equation of the Plane through the point (4,4, 0) and Perpendicular to each of the Planes $x + 2y + 2z - 5 = 0$ and $3x + 3y + 2z - 8 = 0$.

2. Find the equations of the planes bisecting the angle between the planes the equation $x + 2y + 2z = 19$, $4x - 3y + 12z + 3 = 0$.

3. Find the foot of the perpendicular from (1, 2, 3) to the plane $x + 2y + 3z + 4 = 0$.

4. Find the equations of the line through the point (1, 0, −1) and intersecting the lines $4x - y - 13 = 0 = 3y - 4z - 1$; $y - 2z + 2 = 0 = x - 5$.

5. A Plane Passes through a fixed point (a, b, c) and intersects the axes in A, B, C. Show that the Centre of the Sphere OABC lies on $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$.

6. Find the equations of the tangent planes to the sphere $x^2 + y^2 + z^2 - 2x - 4y + 6z - 7 = 0$ which intersect in the line $6x - 3y - 23 = 0 = 3z + 2$.

7. If r_1, r_2 are the radii of the two orthogonal Spheres, then the radius of the Circle of their intersection is $\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$.

8. Find the equation of the quadric cone which passes through the coordinate axes and the three lines $\frac{x}{3} = \frac{y}{5} = \frac{z}{1}$, $\frac{x}{1} = \frac{y}{-1} = \frac{z}{2}$ and $\frac{x}{-11} = \frac{y}{5} = \frac{z}{8}$.

9. Find the enveloping Cone of the Sphere $x^2 + y^2 + z^2 + 2x - 2y = 2$, with its vertex at (1, 1, 1).

10. Find the equation to right circular cone with vertex $P(2, -3, 5)$ axis PQ which makes equal angles with axis and which passes through $A(1, -2, 3)$.

SECTION B — (5 × 10 = 50 marks)

Answer ALL questions.

Each question carries 10 marks.

11. A variable plane is at a constant distance p from the origin and meets the axes in A, B and C. Show that the locus of the centroid of the triangle ABC is $x^{-2} + y^{-2} + z^{-2} = 16p^{-2}$.

Or

12. Show that the equation $x^2 + 4y^2 + 4z^2 + 4xy + 8yz + 4zx - 2x - 10y - 10z + 13 = 0$ represents a pair of parallel planes and find the distance between them.

13. Prove that the lines $\frac{x+1}{1} = \frac{y+1}{2} = \frac{z+1}{3}$, $x + 2y + 3z - 8 = 0 = 2x + 3y + 4z - 11$ are co-planar and their point of intersection.

Or

14. Find the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$, $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$. Find also the equations of shortest distance.

15. Find the centre and radius of the circle $x^2 + y^2 + z^2 - 2y - 4z - 11 = 0$, $x + 2y + 2z - 15 = 0$.

Or

16. Find the equations of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z - 8 = 0$ is a great circle.

17. Find the limiting Points of the coaxial system of Spheres determined by the spheres $x^2 + y^2 + z^2 + 3x - 2y + 6 = 0$, $x^2 + y^2 + z^2 - 4y - 6z + 6 = 0$.

Or

18. If $(-2, 1, -1)$ is a limiting point of the coaxial system for which $x + y + 2z = 0$ is radical plane, then show that the other limiting point is $(-1, 2, -1)$.

19. Find the vertex of the Cone $7x^2 + 2y^2 + 2z^2 - 10xz + 10yz + 2kx - 2y + 2z - 17 = 0$.

Or

20. Find the equations to the right circle cone whose vertex is P $(2, -3, 3)$, axis PQ which makes equal angles with the axis and which passes through A $(1, -2, 3)$.